

Spherical gravitational collapse in 5D Einstein-Gauss-Bonnet gravity

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Abstract. We consider a spherical gravitational collapse of inhomogeneous dust (and null dust) in Einstein gravity with the Gauss-Bonnet (GB) combination of quadratic curvature terms. It turns out that the presence of the coupling constant of the GB terms $\alpha > 0$ completely changes the causal structure of the singularities from the analogous general relativistic case. The gravitational collapse of inhomogeneous dust in the five-dimensional GB extended Einstein equations leads to formation of a massive, but weak, timelike singularity which is forbidden in general relativity. The apparent horizons of two different collapsing solutions show interesting mathematical similarity.

1. Introduction

The Lemaitre-Tolman-Bondi (LTB) [1] and Vaidya [2] solutions have been extensively used not only to study the formation of naked singularities and black holes in spherical collapse, but in cosmology as well. It is well known that these two solutions admit both naked and covered singularities depending upon the choice of initial data and there is a smooth transition from one phase to the other [4]. These results have led to strong evidence against the cosmic censorship conjecture (CCC) [3], which asserts that *there can be no singularity visible from future null infinity*. In recent years a renewed interest has grown in higher order gravity, which involves higher derivative curvature terms, and amongst the most extensively studied theory is the so-called Einstein-Gauss-Bonnet (EGB) gravity. It appears naturally in the low-energy effective action of heterotic string theory [5]. Boulware and Deser [6] found exact black hole solutions in $D (\geq 5)$ -dimensional gravitational theories with a Gauss-Bonnet (GB) term modifying the usual Einstein-Hilbert action. Recently, Maeda [7] considered the spherically symmetric gravitational collapse of a inhomogeneous dust with the $D (\geq 5)$ -dimensional action including the GB term. He investigated its effects on the final fate of gravitational collapse without finding the explicit form of the solution.

In this paper, we consider the 5-dimensional (5D) action with the GB terms for gravity and give an *exact model* of the gravitational collapse of a inhomogeneous dust including the second order perturbative effects of quantum gravity. We also consider the effects of GB terms on the structure of apparent horizons in dust solutions as well as in null dust solutions. The nature of singularities of such a spacetime in terms of its being hidden within a black hole, or whether it would be visible to outside observers, and the consequence of GB term on 5D-LTB collapse are analyzed.

2. Basic equations in EGB

The gravitational part of the 5D action that we consider is [8, 9]:

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa_5^2} (R - 2\Lambda + \alpha L_{GB}) \right] + S_{\text{matter}}, \quad (1)$$

where R and Λ are the 5D Ricci scalar and the cosmological constant respectively. $\kappa_5 \equiv \sqrt{8\pi G_5}$, where G_5 is the 5D gravitational constant. The GB Lagrangian is the combination of the Ricci scalar, Ricci tensor R_{ab} , and Riemann tensor $R^a{}_{b\rho\sigma}$ as $L_{GB} = R^2 - 4R_{ab}R^{ab} + R_{ab\rho\sigma}R^{ab\rho\sigma}$. In the 4-dimensional spacetime, the GB terms do not contribute to the field equations. The α is the coupling constant of the GB terms. From the action (1) we derive the following field equations:

$$\begin{aligned} G_{ab} - \alpha H_{ab} &= T_{ab}, \text{ with } G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R, \\ H_{ab} &= 2 \left[R R_{ab} - 2R_{a\alpha}R_b^\alpha - 2R^{\alpha\beta}R_{\alpha ab\beta} + R_a^{\alpha\beta\gamma}R_{b\alpha\beta\gamma} \right] - \frac{1}{2}g_{ab}L_{GB}, \end{aligned} \quad (2)$$

where G_{ab} is the Einstein tensor and H_{ab} is the Lanczos tensor.

3. Null dust collapse

The energy-momentum tensor of a null fluid is $T_{ab} = \psi(v, r)l_a l_b$, where $\psi(v, r)$ is the non-zero energy density and l_a is a null vector such that $l_a = \delta_a^0$, with $l_a l^a = 0$. Expressed in terms of Eddington advanced time coordinate (ingoing coordinate) v , the metric of general spherically symmetric spacetime [8, 9]:

$$ds^2 = -A(v, r)^2 f(v, r) dv^2 + 2A(v, r) dv dr + r^2 d\Omega_3^2, \quad (3)$$

$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2$. Here $A(v, r)$ is an arbitrary function. It is the field equation $G_1^0 = 0$ that leads to $A(v, r) = g(v)$. This could be absorbed by writing $d\tilde{v} = g(v)dv$. Then, $f(v, r)$ is obtained by solving field equation to give the general solution as

$$f(v, r) = 1 + \frac{r^2}{4\alpha} \left[1 \pm \sqrt{1 + \frac{8\alpha m(v)}{r^4}} \right]; \quad \psi(v, r) = \frac{3}{2r^3} \dot{m}(v), \quad (4)$$

where $m(v)$ is an arbitrary function of v . There are two families of solutions which correspond to the sign in front of the square root in Eq. (4), which has the minus (plus) sign respectively for the minus- (plus+) branch solutions. The energy density of the null fluid as is given by $\psi(v, r)$ for both branches, where the dot denotes the derivative with respect to v . In the general relativistic limit $\alpha \rightarrow 0$, the minus-branch solution reduces to the 5D Vaidya solution in Einstein gravity. In the static case $\dot{m} = 0$, this solution reduces to the solution which was independently discovered by Boulware and Deser [6] and Wheeler [10]. It has been shown that a timelike naked singularity is formed from gravitational collapse of null dust in EGB, which does not appear in the general relativistic case [9].

The apparent horizon (AH) is the outermost marginally trapped surface for the outgoing photons. The AH are defined as surface such that $\Theta \simeq 0$ which implies that $f = 0$. It is clear that AH [8] is the solution of

$$\left[1 + \frac{r^2}{4\alpha} \left[1 - \sqrt{1 + 8\alpha \left(\frac{m(v)}{r^4} \right)} \right] \right] = 0, \text{ implies } r_{AH} = \sqrt{m(v) - 2\alpha}. \quad (5)$$

In the relativistic limit $\alpha \rightarrow 0$ then $r_{AH} \rightarrow \sqrt{m(v)}$. One sees that $g_{vv} = 0$ implies that $r = \sqrt{m(v) - 2\alpha}$ is timelike surface [8].

4. Inhomogeneous dust collapse

The solution we seek is collapse of a spherical dust in 5D-EGB. The energy-momentum tensor for dust is $T_{ab} = \epsilon(t, r)\delta_a^t\delta_b^t$ where $u_a = \delta_t^a$ is the 5D velocity. The metric for the 5D case, in comoving coordinates, is [11, 12, 13]:

$$ds^2 = -dt^2 + A(t, r)^2 dr^2 + R(t, r)^2 d\Omega_3^2. \quad (6)$$

The coordinate r is the comoving radial coordinate, t is the proper time of freely falling shells, R is a function of t and r with $R \geq 0$, and A is also a function of t and r . The master equation of the system is given by

$$\dot{R}^2 \left[1 - 4\alpha \frac{W^2 - 1}{R^2} \right] = (W^2 - 1) + \frac{F}{R^2} - 2\alpha \frac{\dot{R}^4}{R^2}. \quad (7)$$

Here $F = F(r)$ is an arbitrary function of r and is referred to as mass function. Indeed the energy density, $\epsilon = 3F'/2R^3R'$, must be non-negative. It is easy to see that as $\alpha \rightarrow 0$ the master solution (7) of the system reduces to the corresponding 5D-LTB solution in [13]. In the present discussion, we are concerned with gravitational collapse, which requires $\dot{R}(t, r) < 0$. Eq. (7), for $W(r) = 1$, can be integrated to [11]

$$t_\zeta(r) - t = \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{3R^2 - \sqrt{R^4 + 8\alpha F}}{2\sqrt{2}R[\sqrt{R^4 + 8\alpha F} - R^2]^{1/2}} \right] + \sqrt{\frac{\alpha R^2}{\sqrt{R^4 + 8\alpha F} - R^2}}, \quad (8)$$

where $t_\zeta(r)$ is an arbitrary function of integration. The mass function F can be related with initial data (density) at the scaling surface, $t = 0$ ($R = r$), reduces to form $F(r) = \frac{2}{3} \int_0^r \epsilon(0, r) r^3 dr$, which completely specifies the mass function in terms of the initial density profile. The function F must be positive, because $F < 0$ implies the existence of negative mass. This can be seen from the mass function $m(t, r)$ [13], which in the 5D-LTB-EGB case is given by

$$m(t, r) = R^2 \left(1 - g^{ab} R_{,a} R_{,b} \right) = R^2 \left(1 - \frac{R'^2}{A^2} + \dot{R}^2 \right). \quad (9)$$

Using Eq. (7) into Eq. (9), we get $m(t, r) = F(r) - 2\alpha\dot{R}^4$. It may be noted that one can also calculate mass using the formula proposed by Maeda [7] for the generalized mass function in the EGB. The mass function $F(r) = m(t, r) + 2\alpha\dot{R}^4$, is equivalent, up to a constant factor, to the generalized mass function in EGB [7].

The main advantage of working with the AH is that it is local in time and can be located at a given spacelike hypersurface. Considering Eq. (7), the AH condition [11] becomes

$$R(t_{AH}(r), r) = \sqrt{F(r) - 2\alpha}. \quad (10)$$

It is clear that the presence of the coupling constant of the GB terms α produces a change in the location of these horizons. In the relativistic limit, $\alpha \rightarrow 0$, $R_{AH} \rightarrow \sqrt{F(r)}$ [12]. For nonzero α , the structure of the AH is non-trivial. Interestingly, the theory demands α to be a positive number which forbids AH from reaching the center, thereby making the singularity massive and eternally visible, which is forbidden in the corresponding general relativistic scenario. In general relativity, noncentral singularity is always covered [14] (see also [15]). However, in the presence of the GB term we find that even the noncentral singularity is naked, in spite of being massive ($[F(r > 0)] > 0$). Further, Eq. (10) has a mathematical similarity for the AH given by Eq. (5) in null fluid collapse [8].

Equation (10) implicitly defines a curve $t_{ah}(r)$ and represents the AH, i.e. the time at which the shell gets trapped. To further analyze the horizon curve, We use singularity condition in Eq. (8) to get

$$t_c(r) - t = \frac{\pi\sqrt{\alpha}}{4\sqrt{2}} + \sqrt{\frac{\alpha R^2}{\sqrt{R^4 + 8\alpha F} - R^2}} + \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{3R^2 - \sqrt{R^4 + 8\alpha F}}{2\sqrt{2}R[\sqrt{R^4 + 8\alpha F} - R^2]^{1/2}} \right]. \quad (11)$$

Then, the AH condition (10) reduces Eq. (11) to form

$$t_c(r) - t_{AH}(r) = \frac{\pi\sqrt{\alpha}}{4\sqrt{2}} + \frac{\sqrt{\alpha}}{2\sqrt{2}} \tan^{-1} \left[\frac{F - 4\alpha}{2\sqrt{2\alpha(F - 2\alpha)}} \right] + \frac{1}{2} \sqrt{F - 2\alpha}, \quad (12)$$

Clearly, for a positive α , the central shell doesn't get trapped, and the untrapped region around the center increases with increasing α , for both homogeneous and inhomogeneous models. Thus the singularities are always naked as they are formed prior to the formation of apparent horizon.

5. Discussion

In this paper, we have found exact spherically symmetric 5D-LTB solutions, for $W(r) = 1$ case, to GB extended Einstein equations namely 5D-LTB-EGB, which describes gravitational collapse of spherically symmetric inhomogeneous dust in a 5D spacetime. While there may be an AH about this singularity, for $\alpha > 0$, the singularity always remains visible to any observer as the AH lies beyond singularity which is actually not in the spacetime. It is interesting that the coupling constant of the GB terms produces a change in the location of the AH by the factor 2α is exactly the same as in the case of 5D null fluid collapse in EGB. The final fate of gravitational collapse is quite different in the sense that a massive naked singularity is formed, which is disallowed in 5D-LTB. However, the strength singularity is weaker as compared to the corresponding 5D-LTB [11], this may not be a serious threat to CCC. It is seen that the time for the occurrence of the central shell focusing singularity for the collapse is increased as compared to the analogous 5D-LTB case. The reason may be, there is relatively less mass-energy collapsing in the 5D-LTB-EGB spacetime as compared to the 5D-LTB case. In particular, our results in the limit $\alpha \rightarrow 0$ reduce *vis-à-vis* to 5D relativistic case.

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